

THE FIRST $J=1+ T=0$ STATES in a SINGLE j SHELL CONFIGURATION in EVEV-EVEN NUCLEI.

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Abstract

The first even-even nucleus for which there are $J=1+ T=0$ states in a single j shell configuration is ^{48}Cr . If we limit

ourselves to single j there are no M1 transitions from these states to any $J=0+ T=0$ states.

1. Absence of Spin $J=1$ states in j^4 Configurations

1a. $J=1 T=2$ States

1. Introduction

In early shell model calculations by McCullen et al.[1] and Ginocchio and French [2]. it was noted that in the $f_{7/2}$ shell

certain combinations spin and isospin did not exist. For example there were no $J=0+T=1$ states in ^{44}Ti and no $J=1+$

states with $T= T(\text{min})+1$ where $T(\text{min})= |N-Z|/2$. There were also no $J=1+$ states with $T=T(\text{max})$. However those states

are analogous to states of a system of identical particles i.e. calcium isotopes . Explanations for some of the missing

states can be have been shown by low brow techniques as will be discussed later.

In the mid eighties papers were published which counted the states in a more systematic way. There include the works

of I. Talmi on recursion relations for counting the states [3] of identical fermions and by Zhao and Arima [4] who

obtained expressions for the number of $T=0, 1$ and 2 states for protons and neutrons in a single j shell. Indeed the latter

authors give all the answers to the counting questions addressed in this paper.

2.. The first occurrence of $J=1+ T=0$ states in the single j shell- ^{48}Cr

There are no $J=1+ T=0$ states for 4 nucleons in the single j shell. To make things more concrete consider ^{44}Ti . The two $f_{7/2}$ protons can have angular momenta 0,2,4 and 6 all occurring once; likewise the 2 neutrons. The (J_p, J_n) configurations that can add up to a total $J=1$ are (2,2), (4,4) and (6,6). Thus there are three $J=1+$ states. The possible isospins are 0, 1 and 2. Let us next consider ^{44}Sc . The 3 neutrons can have angular momenta $3/2, 5/2, 7/2, 9/2, 11/2$, and $15/2$ all occurring only once. The states that add up to one are $(7/2, 5/2)$, $(7/2, 7/2)$ and $(7/2, 9/2)$. Again we have three states. However since ^{44}Sc has $|T_z|=1$ the isospins can only be one or two. Hence there are no $T=0 J=1+$ states in ^{44}Ti which are of the $(f_{7/2})^4$ configuration.

We next consider ^{48}Cr . The possible states of four protons , including seniority labels are:

$v=0$ $J=0$
 $v=2$ $J=2, 4, 6$
 $v=4$ $J=2^*, 4^*, 5, 8$

The possible $J=1+$ states are $(2,2), (4,4), (6,6), (2^*,2^*), (4^*,4^*), (5,5), (8,8), (2,2^*), (2^*,2), (4,4^*), (4^*,4), (4,5), (5,4), (4^*,5), (5,4^*), (5,6), (6,5)$. There are seventeen such states with a priori possible isospins $T=0,1,2,3,4$. We next consider $48V$ which consists of three protons and five neutrons. The latter can also be regarded as three neutron holes so the possible states are the same for neutrons and protons. The three proton states are $3/2, 5/2, 7/2, 9/2, 11/2$, and $15/2$ all occurring only once. The possible $1+$ states are $(3/2, 3/2), (5/2, 5/2), (7/2, 7/2), (9/2, 9/2), (11/2, 11/2), (15/2, 15/2), (3/2, 5/2), (5/2, 3/2), (5/2, 7/2), (7/2, 5/2), (7/2, 9/2), (9/2, 7/2), (9/2, 11/2), (11/2, 9/2)$. There are 14 such states and they all must have isospins greater than zero. Hence the number of $T=0$ $J=1+$ states of the $(f_{7/2})^8$ configuration is $(17-14)=$ three.

The wave functions of these states is included in a larger compilation by A. Escuderos, L. Zamick and B.F. Bayman [5]

It is there noted that because the both protons and neutrons are at mid shell, the quantity $s = (-1)^V$ is a good quantum number where $V = (v_p + v_n)/2$. Referring to ref [1] for $J=0+ T=0$ there are 4 states with $S=+1$ and two with $S=-1$. All $J=0+ T=1$ states have $s=-1$ while all $T=2$ and $T=4$ states have $s=+1$. There are two $J=1+ T=0$ states with $s=-1$ at energies of 7.775 and 9.258 MeV. There is one $s=+1$ state at 9.037 MeV with a rather simple wave function $1/\sqrt{2} [(4^*,5) + (5,4^*)]$. The lowest $J=0+ T=0$ state has $s=+1$.

4. M1 Selection Rules

There is a modern twist to what we are here doing. There has been an extensive review of M1 excitations, including spin flip modes, scissors modes e.t.c. by K. Heyde, P. Von Neumann-Cosel and A. Richter [6]. The mode we are here considering, has, to best of our knowledge, not yet been studied experimentally. There have been studies of M1 $T=0$ to $T=0$ transitions e.g. the electro-excitation of $T=0$ $J=1+$ excited states of ^{12}C but these involve more than one shell [7]. Isospin impurities are very important for these transitions because the isovector M1 coupling constants are much larger than the isoscalar ones.

One simple selection rule for M1 transitions in this limited model space is that $M1(T=0 \rightarrow T=0)$ equals zero.

To see this we note that in the single j shell space we can replace the M1 operator by $g_j J$. The M1 matrix element for a $T=0$ to $T=0$ transition is thus proportional to $(g_{j\pi} + g_{j\nu})$, i.e. the isoscalar sum. But if such a term is non-zero it would imply that the total angular momentum operator J (obtained by setting the two g 's above each equal to $1/2$) could induce an M1 transition, which, of course, it cannot.

Another "midshell" selection rule is that the quantum number s has to be the same for the initial $J=1, T=0$ state and for

any final state e.g. $J=1+, T=1$ or $J=2+, T=1$.

Although not necessary it is nevertheless instructive to show in more detail why the $T=0$ to $T=0$ matrix element vanishes. Consider a transition from $s=-1$

to $s=-1$. In the wave functions there will be no amplitude of the configuration $(J_p, J_n) = (2, 2)$ but there will be of $(2, 2^*)$ and $(2^*, 2)$. The transition matrix element will have the form

$$\langle (2, 2^*)^2 + (2^*, 2)^2 || M1 || (2, 2^*)^1 - (2^*, 2)^1 \rangle. \text{ This is equal to } \langle (2, 2^*)^2 || M1 || (2, 2^*)^1 \rangle - \langle (2^*, 2)^2 || M1 || (2^*, 2)^1 \rangle$$

. Since, in the single j shell one can replace $M1$ by $g_j J$ the matrix element $\langle 2 || M1 || 2 \rangle$ is equal to $\langle 2^* || M1 || 2^* \rangle$

. We thus see that the complete matrix element vanishes.

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